Bivariate return periods and their importance for flood peak and volume estimation

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Abstract: Estimates of flood event magnitudes with a certain return period are required for the design of hydraulic structures. While the return period is clearly defined in a univariate context, its definition is more challenging when the problem at hand requires considering the dependence between two or more variables in a multivariate framework. Several ways of defining a multivariate return period have been proposed in the literature, which all rely on different probability concepts. Definitions use the conditional probability, the joint probability, or can be based on the Kendall's distribution or survival function. In this study, we give a comprehensive overview on the tools that are available to define a return period in a multivariate context. We especially address engineers, practitioners, and people who are new to the topic and provide them with an accessible introduction to the topic. We outline the theoretical background that is needed when one is in a multivariate setting and present the reader with different definitions for a bivariate return period. Here, we focus on flood events and the different probability concepts are explained with a pedagogical, illustrative example of a flood event characterized by the two variables peak discharge and flood volume. The choice of the return period has an important effect on the magnitude of the design variable quantiles, which is illustrated with a case study in Switzerland. However, this choice is not arbitrary and depends on the problem at hand.

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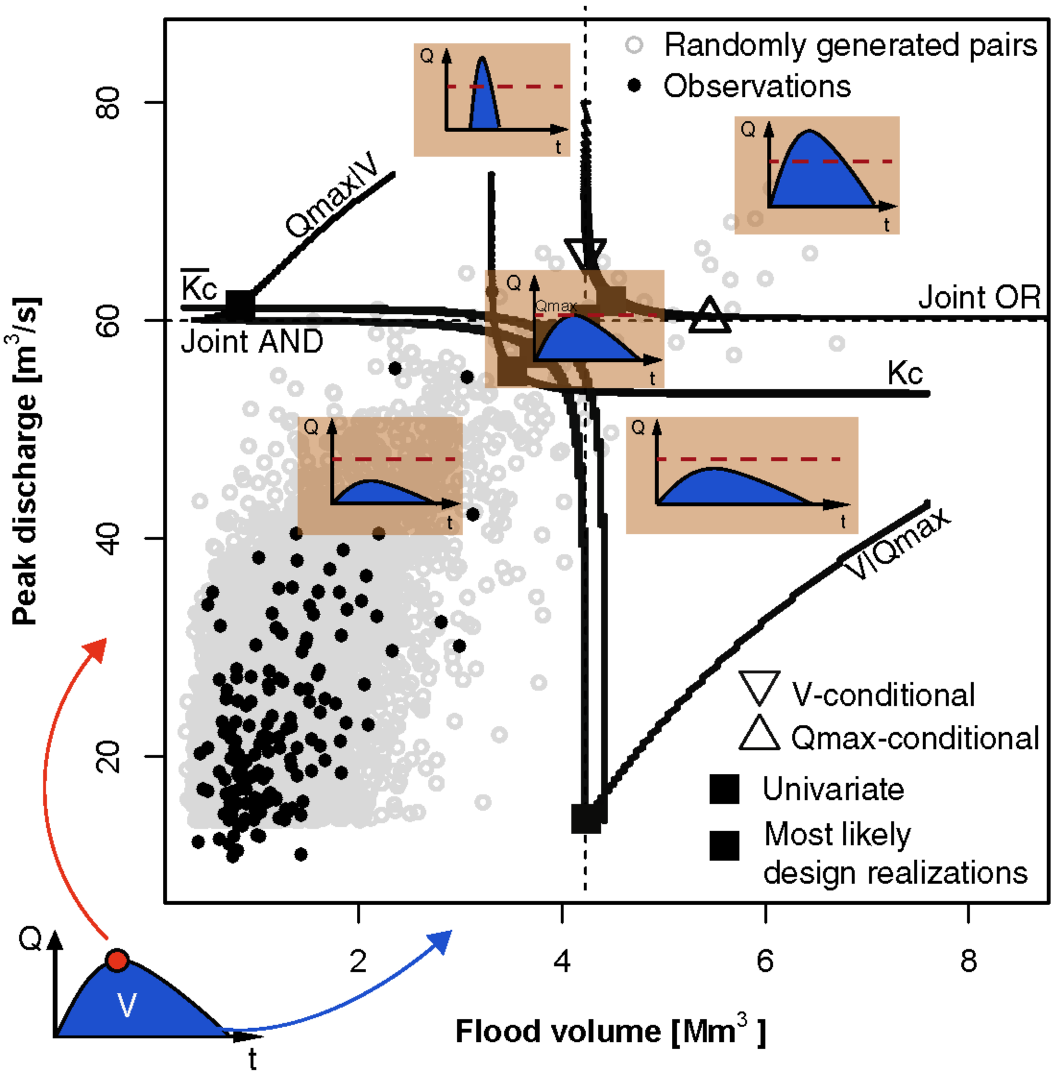
ABSTRACT

Estimates of flood event magnitudes with a certain return period are required for the design of hydraulic structures. While the return period is clearly defined in a univariate context, its definition is more challenging when the problem at hand requires considering the dependence between two or more variables in a multivariate framework. Several ways of defining a multivariate return period have been proposed in the literature, which all rely on different probability concepts. Definitions use the conditional probability, the joint probability, or can be based on the Kendall's distribution or survival function. In this paper, we give a comprehensive overview on the tools that are available to define a return period in a multivariate context. We especially address engineers, practitioners, and people who are new to the topic and provide them with an accessible introduction to the topic. We outline the theoretical background that is needed when one is in a multivariate setting and present the reader with different definitions for a bivariate return period. Here, we focus on flood events and the different probability concepts are explained with a pedagogical, illustrative example of a flood event characterized by the two variables peak discharge and flood volume. The choice of the return period has an important effect on the magnitude of the design variable quantiles, which is illustrated with a case study in Switzerland. However, this choice is not arbitrary and depends on the problem at hand.

KEYWORDS

Bivariate return period, joint probability, conditional probability, return period definition, flood estimates, hydraulic design

GRAPHICAL ABSTRACT



Graphical abstract: When both flood peak and volume are of interest, different return period definitions are possible which result in different design variables.

INTRODUCTION

The design of hydraulic structures requires reasonable estimates for flood events that have a certain likelihood of occurrence in the catchment under consideration. These estimates are called design variables and are usually quantified for a given return period [1]. The return period is defined as the average occurrence interval which refers to the expected value of the number of realizations to be awaited before observing an event whose magnitude exceeds a defined threshold [2, 3]. This definition is valid under the assumption that the phenomenon is stationary over time and each realization is independent of the previous ones [2]. The return period provides a simple, yet efficient means for risk assessment because it concentrates a large amount of information into a single number. More probable events have shorter return periods, less probable events have longer return periods [4].

In engineering practice, the choice of the return period depends on the importance of the structure under consideration and the consequences of its failure [5]. National laws and guidelines usually fix a return period for dam design. However, they do not specify whether it refers to the peak discharge, the flood volume, or the entire hydrograph [6]. Strictly, a T-year hydrograph does not exist. All hydrographs are different and a frequency can only be ascribed to a particular aspect of a hydrograph, such as its peak flow, its volume, or to a particular impact such as the level of inundation [7]. However, hydrological events are not only described by one variable but by a set of correlated random variables usually consisting of the flood peak, flood volume, and duration. If more than one of these variables is significant in the design process, a univariate frequency analysis, where only one variable is considered, *e.g.* the peak discharge, can therefore not provide a complete assessment of the probability of occurrence of a flood event [8] and might lead to an inappropriate estimation of the risk associated with that event [4]. An overestimation of the risk is not desirable because it will increase the costs of constructing the hydraulic structure. Estimating too low design values might be even worse because it increases the risk of failure.

If two or more design variables, which are not independent from each other, are significant in the design process, one needs to consider the dependence between these variables when doing flood frequency analysis. It was shown that in such a case, a bi- or multivariate analysis where two or more variables are considered, *e.g.* peak discharge and flood volume, will lead to more appropriate estimates than a univariate analysis [1, 4, 8]. The problem of how to define a return period in a multivariate context has been addressed in several publications over the last 15 years. Several ways of defining a multivariate return period have been proposed which rely on different probability concepts. Definitions use the conditional probability, the joint probability, or can be based on the Kendall's distribution or survival function [1, 4, 5, 8-12]. The choice of a definition for a multivariate return period is not arbitrary and depends on the problem at hand [2].

Therefore, the goal of this paper is not to present the definitive definition of a multivariate return period but to give a comprehensive overview of the tools that are available to define a return period in a multivariate context. We describe the definitions that have been proposed in previous publications, expressing them with and without copulas, and illustrate them with a practical example. This overview especially addresses engineers, practitioners, and people who are new to the topic and gives them an accessible introduction to the topic by providing the background for deciding on suitable strategies of defining a return period for a particular application. Important issues that need to be addressed when wanting to estimate design variables for a certain return period are discussed.

We first provide the reader with the theoretical background that is needed when one is dealing with return periods in a multivariate setting. Then, we outline several ways of defining a bivariate return period. We provide equations only if we think that this can clarify the situation and support understanding. Following the common notation, we use upper case letters for random variables or events and lower case letters for values, parameters, or constants. Throughout this paper, we use the example of a flood event characterized by the two design variables, peak discharge and flood volume, which are illustrated in Figure 1.

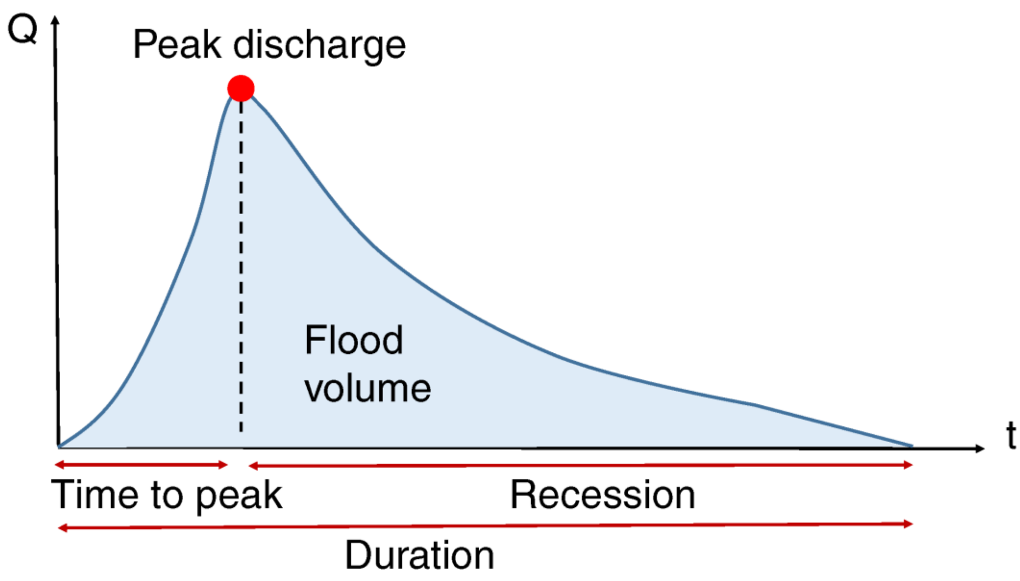


Figure 1: Illustration of different flood hydrograph characteristics.

A third potential variable would be the flood duration, which would add a third dimension to the analysis and move us to a trivariate setting. For simplicity, we compute the volume of an event always over a window of 72 hours and thereby keep the duration constant. This allows us to focus on bivariate return periods, which makes calculations less complex. However, the tools presented here are also applicable in more than two dimensions.

After a more theoretical part on return periods, the influence of the choice of a specific bivariate return period, made *a priori* according to the problem at hand, on the design variables is illustrated on a case study using data from the Birse catchment at Moutier-la-Charrue in Switzerland.

BACKGROUND

Practices in estimating design variables

When estimating design variables for a hydraulic structure, we usually talk about design variable quantiles. The quantile can be defined as the magnitude of the event in terms of its non-exceedance probability [13]. If one considers the p-quantile, values in the sample have a probability of p% of not exceeding this quantile. The information of the non-exceedance probability is contained in the return period. The return period is used in national guidelines to define levels of flood protection and rules for the construction of hydraulic structures. These guidelines differ from country to country but they have in common that areas and structures of lower importance are protected against events with lower return periods while inhabited areas and critical structures are protected against events with higher return periods [14]. In Switzerland, for example, flood protection goals for agricultural land and infrastructure are based on a 20-year flood, *i.e*. a flood with a return period of 20 years, and protection goals for inhabited areas on a 100-year flood [15]. Very sensitive structures such as dams built for the storage of water for hydropower production have even higher protection goals. Usually, protection goals for such critical structures are based on events of a return period between 500 to 10 000-years depending on the type of the dam [6].

Definition of a univariate return period

We need to define the univariate return period before dealing with bivariate return periods. The value of the cumulative distribution function F_X of a random variable X at a given value x is the probability that the random variable X is less than or equal to x

$$F_X(x) = \Pr[X \leq x] \tag{1}$$

In hydrology, we would for example talk about the probability that the peak magnitude of a certain flood event, here denoted by X, is smaller than a given runoff threshold, here denoted by x.

In contrast, the exceedance probability that x will be equaled or exceeded is given by the survival function S_X of the random variable X, which is often used in statistical literature and stands for

$$S_X(x) = 1 - F_X(x). \tag{2}$$

If we consider our hydrological example again, we talk about the probability that the peak magnitude X of a certain event exceeds a given runoff threshold x.

The return period $T(x)$ of the event $\{X \geq x\}$ can be written as

$$T(x) = \frac{\mu}{S_X(x)} = \frac{\mu}{1 - F_X(x)}, \tag{3}$$

where μ is the mean inter-arrival time between two successive events, which is defined as one divided by the number of flood occurrences per year [8]. If we look at annual maxima, μ corresponds to 1 year. In our example, $T(x)$ stands for the (univariate) return period of an event where the peak magnitude X exceeds the threshold x.

The definition of a univariate return period can be expressed as one single equation. In practice, however, one is often faced with problems where two variables are important in the design process. For example, we often not only need to consider the flood peak, but also the flood volume. If the two variables depend on each other, we need to take into account their dependence. For this, we can look at their conditional probability of occurrence, their joint probability of occurrence or work with the Kendall's distribution or survival function. The choice of one of these probability concepts depends on the application under consideration.

Even in a bivariate context, the marginal distributions, *i.e.* the distributions of the single variables independent of the other variables, are of great interest. We need to analyze the marginal distributions of the design variables peak discharge and flood volume before having a look at their conditional or joint distribution.

Marginal distributions of design variables

The marginal distributions of our variables peak discharge and flood volume are linked to how we sample flood events. There are two main approaches to choose flood events from a runoff time series. The first one is the block maxima approach, which is based on choosing the highest event (usually looking at the peak discharges) over a period of time. The second approach is the peak-over-threshold approach (POT), which is based on choosing all peaks that lie above a predefined threshold. While the block maxima approach, in which the block is defined as a year, retains only one event per year, it is possible to choose more than one event per year using the POT approach depending on the choice of the threshold [16]. After the sampling with one of these two approaches, we have a series of flood events characterized by the variables peak discharge and flood volume. Extreme value theory [17] says that block maxima follow a generalized extreme value (GEV) distribution while peak-over-threshold series follow a generalized Pareto distribution (GPD). The GEV model has three continuous parameters: a location parameter $\mu_l \in \mathbb{R}$, a scale parameter $\sigma > 0$, and a shape parameter $\xi \in \mathbb{R}$ and is defined as

$$F_X = \exp\left[-\left\{1+\xi\left(\frac{x-\mu_l}{\sigma}\right)\right\}^{-\frac{1}{\xi}}\right] \qquad \xi \neq 0, \tag{4}$$

defined on $\left[\xi : \left\{1+\xi\left(\frac{x-\mu_l}{\sigma}\right)\right\} > 0\right]$.

On the other hand, the GPD uses the same parameters and is expressed as

$$F_X = 1-\left\{1+\xi\left(\frac{x-\mu_l}{\sigma}\right)\right\}^{-\frac{1}{\xi}} \qquad \xi \neq 0, \tag{5}$$

defined on $\left[x-\mu_l : \{x-\mu_l\} > 0 \text{ and } \left\{1+\xi\left(\frac{x-\mu_l}{\sigma}\right)\right\} > 0\right]$. Often, in flood frequency analysis, one works with annual maxima to guarantee the independence of the events analyzed. However, the disadvantage is that some important events are neglected because only the highest event per year is included in the data set. This problem can be solved by using a POT approach. However, even though the choice of the threshold is crucial, it is somewhat subjective [17].

Modelling the dependence between two or more variables

Once we defined the marginal distributions of our variables, we need to study their relationship and to assess the strength of their dependence [18]. If there is no dependence between two variables, their joint distribution is simply the product of the marginal distributions. However, if there is any dependence, we have to model their joint behavior. The cumulative distribution function F_{XY} of two variables X and Y allows us to define the probability $F_{XY}(x,y)$ that both X and Y do not exceed given values x and y [19]

$$F_{XY}(x,y) = \Pr[X \leq x, Y \leq y]. \tag{6}$$

Traditionally, the pairwise dependence between variables such as the peak, volume and duration of flood events has been described using classical families of bivariate distributions.

The main limitation of these bivariate distributions is that the individual behavior of the two variables must be characterized by the same parametric family of univariate distributions. Copula models which are multivariate distribution functions avoid this restriction. Recent developments in statistical hydrology have shown the great potential of copulas for the construction of multivariate cumulative distribution functions and for carrying out a multivariate frequency analysis [1, 20]. A list of publications on copula functions and their use in hydrology can be found on the web page of the International Commission on Statistical Hydrology [21] [a].

The copula approach to dependence modeling is rooted in a representation theorem due to Sklar [22]. He stated that the value of the joint cumulative distribution function F_{XY} of any pair (X, Y) of continuous random variables at (x, y) may be written in the form of

$$F_{XY}(x,y) = C\{F_X(x), F_Y(y)\} = C(u,v)\,, x, y \in \mathbb{R}\,, \tag{7}$$

where $F_X(x)$ denoted by u and $F_Y(y)$ denoted by v are realizations of the marginal distributions of X and Y whose dependence is modelled by a copula C. Our attention is restricted to the pair of random variables (U, V), where U denotes $F_X(X)$ and V denotes $F_Y(Y)$. The probability integral transform allows for the conversion of the random variables $F_X(X)$ and $F_Y(Y)$ from the continuous distributions F_X and F_Y to the random variables U and V having a uniform distribution $U(0,1)$. In our example, F_X stands for the marginal distribution of the peak discharge values, F_Y represents the marginal distribution of the flood volume values, and F_{XY} denotes the joint distribution of peak discharges and flood volumes.

Sklar showed that C, F_X, and F_Y are uniquely determined when their joint distribution F_{XY} is known. The selection of an appropriate model for the dependence between X and Y represented by the copula can proceed independently from the choice of the marginal distributions [23]. Copulas are functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions [24]. The large number of copula families proposed in the literature allows one to choose from a large quantity of dependence structures [23].

Five steps are involved in modeling the dependence between two or more variables with a copula:

1. Evaluation of the dependence between the variables doing an exploratory data analysis using K-plots and Chi-plots as well as suitable Kendall's and Spearman's independence tests [23].
2. Choice of a number of copula families.
3. Estimation of copula parameters for each copula family.
4. Exclusion of non-admissible copulas via suitable goodness-of-fit tests [23, 25].
5. Choice of an admissible copula via selection criteria such as the Akaike or Bayesian information criterion [26].

[a] http://www.stahy.org/Topics/CopulaFunction/tabid/67/Default.aspx

For a more thorough introduction to copulas, we refer to the textbooks of Nelsen [24] or Joe [27] or the review paper by Genest and Favre [23]. For an application of copulas to estimate return periods for hydrological events, we refer to the textbook of Salvadori et al. [20].

BIVARIATE RETURN PERIODS

A bivariate analysis is advisable when two dependent variables play a significant role in ruling the behaviour of a flood [12]. In bivariate frequency analysis, in contrast to univariate frequency analysis, the definition of an event with a given return period is not unique [8], however, it is determined by the problem at hand [2]. Salvadori et al. [5] provided a general definition of a return period which does not only apply for the univariate but also a multivariate setting and therewith helps to make the step from a univariate setting to a bivariate or multivariate framework. They defined the return period T_D of a "dangerous" event as

$$T_D = \frac{\mu}{\Pr[X \epsilon D]}, \quad (8)$$

where D is a set collecting all the values judged to be dangerous according to some suitable criterion, μ is the average inter-arrival time of two realizations of X, and $\Pr[X \epsilon D]$ is the probability of a random variable (vector) X to lie in the dangerous region D. In a setting with one significant design variable, a critical design value x is used to identify the dangerous region D consisting of all values exceeding x. In our hydrological example, x would refer to a peak discharge threshold above which an event is considered dangerous. In a bivariate context, the dangerous region D can be defined in various ways allowing for different return period definitions according to the problem at hand. Recently, Salvadori et al. [28] introduced the term "hazard scenario" for a set containing all the occurrences of X said to be dangerous. The ways the term return period is used in the following are all special cases of the definition given in Equation 8.

The return period used to describe bivariate events can be determined by three types of approaches. The first of these approaches uses the conditional probability to determine a conditional return period, while the second method uses joint probability distributions to calculate joint return periods and the third approach relies on the Kendall's distribution or survival function. In hydrology, the conditional probability can for example describe the probability of a peak discharge to exceed a given threshold given that the flood volume exceeds a given threshold, or vice versa. The joint probability distributions can for example describe the following two situations. First, the probability that both the peak discharge and the flood volume exceed certain thresholds during a flood event. Second, the probability that either the peak discharge or the flood volume exceed given thresholds.

The three main approaches to determine a bivariate return period are described in more detail in the next paragraphs.

Conditional return period

The conditional return period approach is typically applied in situations in which one of the design variables is considered to be more important than the other one [12]. The conditional return period relies on a conditional probability distribution function of a variable given that some condition is fulfilled. The conditional return period approach can apply to particular conditional events which are chosen depending on the problem at hand. Here, we focus on two types of events that might be of special interest when designing a hydraulic structure. However, other conditional events could be investigated if necessary. The two events analyzed are described as

$$E^{>}_{X|Y} = \{X > x|\ Y > y\} \text{ and} \tag{9}$$

$$E^{>}_{Y|X} = \{Y > y|\ X > x\}, \tag{10}$$

with associated probability $\Pr\,[X > x|\ Y > y]$ and $\Pr\,[Y > y|\ X > x]$ respectively. Picking up our hydrological example again, event number one corresponds to the situation where the peak discharge X exceeds a threshold x given (denoted as $|$) that the flood volume Y exceeds a threshold y. This event would be used if flood volume was considered to be the crucial variable. Event number two corresponds to the situation where the flood volume exceeds a threshold given that the peak discharge exceeds a predefined threshold. This event would be used if peak discharge was considered to be the most important variable in the design process.

The values of the conditional probability distribution functions for these events are defined as

$$F_{X|Y}(x,y) = 1 - \frac{F_X(x) - F_{XY}(x,y)}{1 - F_Y(y)} \text{ and} \tag{11}$$

$$F_{Y|X}(x,y) = 1 - \frac{F_Y(y) - F_{XY}(x,y)}{1 - F_X(x)}. \tag{12}$$

The conditional return period of these two conditional events can therefore be described as

$$T(x|y) = \frac{\mu}{1 - \frac{F_X(x) - F_{XY}(x,y)}{1 - F_Y(y)}} \text{ and} \tag{13}$$

$$T(y|x) = \frac{\mu}{1 - \frac{F_Y(y) - F_{XY}(x,y)}{1 - F_X(x)}}. \tag{14}$$

The conditional return period describes the mean time interval between two situations of exceedance of a certain flood volume given that a certain flood peak is exceeded or vice versa.

Conditional return period using copulas

The study of conditional distributions can be facilitated using copulas according to Salvadori and De Michele [4], Salvadori [9], Renard and Lang [29], Salvadori et al. [20], Vandenberghe et al. [11], Salvadori and De Michele [10], Durante and Salvadori [30], Salvadori et al. [5], and Gräler et al. [1].

We consider again the two conditional events given in Equations 9 and 10 but work with the random variables U and V which have a uniform distribution and stand for $F_X(X)$ and $F_Y(Y)$. Using copulas, the corresponding conditional return periods are denoted by

$$T(u|v) = \mu\ \frac{1 - v}{1 - u - v + C(u,v)} \text{ and} \tag{15}$$

$$T(v|u) = \mu\ \frac{1 - u}{1 - u - v + C(u,v)}, \tag{16}$$

where μ is the mean inter-arrival time between two sampled flood events.

Joint return period

The joint return period of a multivariate event can be calculated using different joint probability distribution functions. Four different ways of defining values of the joint probability distribution function are illustrated in Figure 2a. Quadrants I to IV show different ways of defining a joint probability:

Quadrant I: $\Pr\,[X > x, Y > y] = 1 - F_X(x) - F_Y(y) + F_{XY}(x, y) = S_{XY}(x, y)$

Quadrant II: $\Pr\,[X \leq x, Y > y]$

Quadrant III : $\Pr\,[X \leq x, Y \leq y] = F_{XY}(x, y)$

Quadrant IV : $\Pr\,[X > x, Y \leq y]$

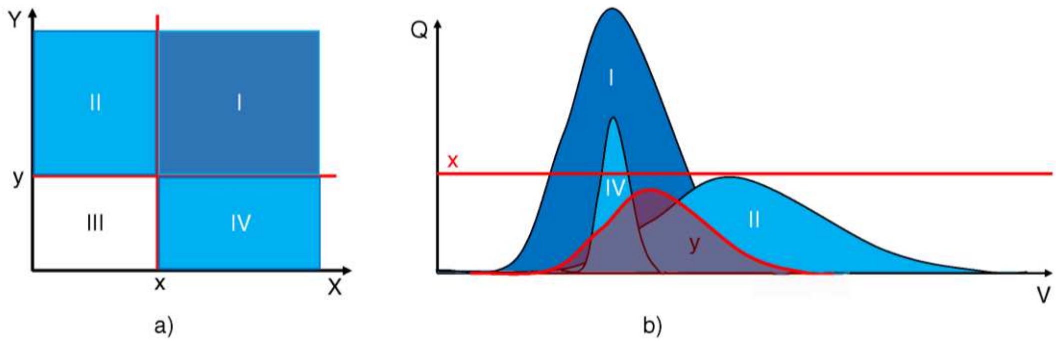


Figure 2: a): Illustration of joint probabilities. Quadrant I shows the case when both variables X and Y exceed the values x and y. Quadrant II shows the case where Y but not X exceeds the reference value. Quadrant III shows the case where neither X nor Y exceed their reference values. Finally, Quadrant IV shows the case when X but not Y exceeds the reference value. The figure was modified after Yue et al. [31]. b): Hydrological example. The red line stands for the peak discharge threshold x, the red hydrograph for the threshold value of total flood volume y. For each quadrant in figure a, one example event is given. The flood event in Quadrant I has a higher peak discharge and a higher flood volume than given by the thresholds. The event in Quadrant II has a higher volume than the threshold but a lower peak discharge. The event in Quadrant IV has a lower volume than the threshold but a higher peak discharge.

In flood frequency analysis, we might either be interested in working with events situated in Quadrant I, where X exceeds x and Y exceeds y, or we want to work with the events situated in Quadrants II and IV where either Y exceeds y or X exceeds x [8]. These possible joint events using the OR and the AND operators, *i.e.*," ∨ " and, *i.e.*, " ∧ " , are given in Table 1 [4, 9].

Table 1: Possible joint events using the OR " ∨ " and the AND " ∧ " operators. Potential events of special interest in flood frequency analysis are highlighted in blue.

OR ∨	$\{\boldsymbol{X \leq x}\}$	$\{\boldsymbol{X > x}\}$
$\{\boldsymbol{Y \leq y}\}$	$\{X \leq x\} \vee \{Y \leq y\}$	$\{X > x\} \vee \{Y \leq y\}$
$\{\boldsymbol{Y} > y\}$	$\{X \leq x\} \vee \{Y > y\}$	$\{X > x\} \vee \{Y > y\}$

AND $\wedge$	$\{\boldsymbol{X} \leq \boldsymbol{x}\}$	$\{\boldsymbol{X} > \boldsymbol{x}\}$
$\{\boldsymbol{Y} \leq \boldsymbol{y}\}$	$\{X \leq x\} \wedge \{Y \leq y\}$	$\{X > x\} \wedge \{Y \leq y\}$
$\{\boldsymbol{Y} > y\}$	$\{X \leq x\} \wedge \{Y > y\}$	$\{X > x\} \wedge \{Y > y\}$

Continuing with our hydrological example (see Figure 2b), the events located in Quadrant I correspond to events where both the peak discharge X and the flood volume Y exceed given thresholds x and y. Events located in Quadrant II correspond to flood events where the flood volume exceeds a given threshold but not the peak discharge. On the contrary, events located in Quadrant IV correspond to flood events where the peak discharge but not the flood volume exceeds a certain threshold.

The return period of events situated in Quadrants I, II or IV where either peak discharge or flood volume (or both) exceeds a given threshold can be expressed by the joint OR return period (Equation 17)

$$T^{\vee}(x,y) = \frac{\mu}{\Pr[X>x \vee Y>y]} = \frac{\mu}{1-F_{XY}(x,y)}. \quad (17)$$

The return period of events situated in Quadrant I where both peak discharge and flood volume exceed a threshold can be expressed as the joint AND return period [6, 8, 32] (Equation 18)

$$T^{\wedge}(x,y) = \frac{\mu}{\Pr[X>x \wedge Y>y]} = \frac{\mu}{1-F_X(x)-F_Y(y)+F_{XY}(x,y)}. \quad (18)$$

Joint return period using copulas

The bivariate joint distribution of flood peak and volume can also be obtained using a bivariate copula model [6]. Thus, the joint distribution function used for the calculation of a return period can be expressed in the form of a copula. For example, let us again consider the two events of particular interest given in Table 1, *i.e.*

$$\{U > u\} \vee \{V > v\} \text{ and} \quad (19)$$

$$\{U > u\} \wedge \{V > v\}, \quad (20)$$

where U stands for $F_X(X)$, the peak discharge transformed via the probability integral transform, and V stands for $F_Y(Y)$, the flood volume transformed via the probability integral transform. In the first event, either the transformed peak U or the transformed volume V does not exceed a certain probability u or v respectively. In the second event, both U and V do not exceed a certain probability u or v. The choice of one of these events depends, as mentioned above, on the problem at hand.

The joint OR and AND return periods of these two events using a copula can be calculated as follows

$$T^{\vee}(u,v) = \frac{\mu}{1-C(u,v)} \text{ and} \quad (21)$$

$$T^{\wedge}(u,v) = \frac{\mu}{1-u-v+C(u,v)}, \quad (22)$$

where μ is the mean inter-arrival time between successive events [9]. The return period only depends on the copula and not on the marginal distributions. These are just used to return from the space defined by the uniform distributions of U and V to the space of the real distributions of X and Y. All pairs (u, v) that are at the same probability level of the copula (*i.e.*, they lie on an isoline of the copula) will have the same bivariate return period.

Kendall's return period

Salvadori and De Michele [10] introduced the Kendall's distribution function (Equation 23), which depends only on the copula function C, and thus partitions the sample space into a super-critical and a non-critical region. The Kendall's distribution function K_c stands for the cumulative distribution function of the copula's level curves or isolines and is given in a bivariate case by

$$K_c(t) = \Pr\,[W \leq t] = \Pr\,[C(U,V) \leq t], \tag{23}$$

where $W = C(U,V)$ is a univariate random variable [5,10]. In the bivariate case, analytical expressions for K_c are available for both Archimedean and Extreme Value copulas [20,33]. When no analytical expression for K_c is available, it needs to be calculated numerically based on a simulation algorithm [5]. The Kendall's distribution function allows for the calculation of the probability that a random pair (U, V) in the unit square has a smaller (or larger) copula value than a given critical probability level t. Any critical probability level t uniquely corresponds to a subdivision of the space into a super-critical and a non-critical region. The Kendall's return period therefore corresponds to the mean inter arrival time of critical events lying on the probability level t which is given by

$$T_{K_c} = \frac{\mu}{1 - K_c(t)}\,. \tag{24}$$

Events more critical than the design event, *i.e.*, the so-called super-critical or dangerous events, have a larger Kendall's return period than the events lying on the critical isoline and will appear much less frequently than the given design return period. This Kendall-based approach ensures that all super-critical events have a longer joint return period than the design value, while some non-critical events might have larger marginal values than any selected design event [1].

To overcome this issue, Salvadori et al. [34] introduced the survival Kendall's return period which yields a bounded safe region, where all the variables of interest are finite and limited. The survival Kendall's return period is based on the survival Kendall's distribution function instead of the Kendall's distribution function and is defined as

$$T_{\overline{K_c}} = \frac{\mu}{1 - \overline{K_c}(t)}, \tag{25}$$

where $\overline{K_c}$is the Kendall's survival function given by

$$\overline{K_c}(t) = \Pr[S_{XY}(X,Y) \geq t] = \Pr\,[C(1-U, 1-V) \geq t] \tag{26}$$

, where S_{XY} is the survival function of X and Y [12,34]. The factor $1 - \overline{K_c}(t)$ yields the probability that a multivariate event will occur in the super-critical region [12].

One of the conditional or joint return period definitions introduced above can be used to estimate design variable quantiles according to the problem at hand. However, these definitions do not take into account any interaction of the design variables peak discharge and flood volume and the hydraulic structure to be designed. To overcome this shortcoming, Volpi and Fiori [35] introduced the structure-based return period which allows for the consideration of the structure in hydraulic design in a bivariate or multivariate environment. The structure-based return period is based on the assumption that the structure design parameter Z is related to the hydrological variables X and Y through a strictly monotonic structure function $Z = g(X, Y)$.

The return period of structure failure T_Z (Equation 27) can be computed by applying a standard univariate frequency analysis to the random variable Z using its distribution function F_Z:

$$T_Z = \frac{\mu}{1-F_Z(z)}, \tag{27}$$

where μ is again the mean inter-arrival time between two successive events. Salvadori et al. [36] stated that it may be awkward and impractical to select the univariate law of F_Z analytically, especially when the structure function is nonlinear. Therefore, they proposed to use Monte Carlo techniques to obtain an approximation of F_Z.

Isolines

The difference between the univariate and the bivariate approach is that in the bivariate case, there is no unique solution of design variables associated with the return period T. Specific conditional or joint return periods can be achieved using various combinations of the two random variables. Hence, the bivariate return period for flood peak and flood volume must be illustrated using contour lines [32]. This return period level is a curve on a bi-dimensional graph with peak discharge and flood volume as coordinates. Based on the contours of the conditional, joint or (survival) Kendall's return periods, one can obtain various combinations of flood peaks and volumes for a given return period [8, 9]. The isolines of the joint OR return period are the level curves of the copula C of interest, while the isolines of the joint AND return period are the level curves of the survival copula of interest. Similarly, the conditional return period is constant over the isolines of the functions defined in Equations 15 and 16.

Choice of a realization on the isoline

In some cases of application, it might be desirable to have just one design realization or a subset of all possible realizations instead of a large set of potential events for a specified return period. Usually, one event on the isoline is chosen and declared as the design event. Several options exist for choosing one or more design realizations from the return level curve. These can be grouped into two classes. The first class of approaches aims at choosing only one design realization, whereas the second class aims at selecting a subset of design realizations on the return level curve.

Salvadori et al. [5] proposed two approaches to choose one design realization. One of these approaches looks for the "component-wise excess design realization" whose marginal components are exceeded with the largest probability. The second approach looks for the "most-likely design realization" taking into account the density of the multivariate distribution of the flood events. The most-likely design realization of all possible events can be obtained by selecting the point with the largest joint probability density [1, 5] using Equation 28

$$(u,v) = \underset{C(u,v)=t}{\operatorname{argmax}} f_{XY}\big(F_X^{-1}(u), F_Y^{-1}(v)\big). \tag{28}$$

These two approaches are just two ways of choosing one design realization. In general, to identify one design realization, a suitable weight function needs to be fixed and the point(s) where it is maximized on the critical layer can be calculated [5, 37]. Salvadori et al. [12] proposed another method to choose one design realization which is applicable if one of the variables (*e.g.* X) is seen as the ruling variable (they called it H-conditional approach because their ruling variable was called H). Here, we would rather talk about the X-conditional or Y-conditional approach. Given a return period T and using a univariate approach, the corresponding critical probability level p can be calculated using Equation 3. Knowing p, the fitted marginal distribution of the ruling variable, F_X, can be inverted to provide us with a design value x_T for the driving variable

$$x_T = F_X{}^{-1}(p). \tag{29}$$

Considering a particular isoline (*e.g.* conditional, joint, or (survival) Kendall's), a design value y_T can be provided for the second variable. This corresponds to the point where the design value x_T of the first variable intersects with the isoline.

The advantage of choosing just one design realization is that it is easy to handle. However, the selection of just one event reduces the amount of information that can be obtained by the multivariate approach. If one wants to keep more of this information and is rather interested in choosing a subset of design realizations from the return level curve, there are also different options. Chebana and Ouarda [13] divided the return level curve into a naïve and a proper part. The naïve part is composed of two segments starting at the end of each extremity of the proper part. The extremities are defined by the maximum values for each of the variables. An alternative is the ensemble approach proposed by Gräler et al. [1]. They suggested to sample across the contours of the return level plot according to the likelihood function. By doing so, the highest density of design events is sampled around the most-likely realization, whereas less design events are sampled on the two outer limits of each contour, corresponding to the naïve part in the approach of Chebana and Ouarda [13]. Once a subset has been selected, a practitioner can still choose one design event from the subset according to the event's effect on the hydraulic structure under consideration [38].

Choice of return period definition

The return period definition to be used in flood frequency analysis should be determined *a priori* according to the hydraulic structure to be designed or the risk assessment problem to be solved [2]. The choice of the most suitable approach to calculate the return period should be evident once the problem at hand is well defined and will affect the calculation of the design event. The different return periods do not provide answers to the same problem statement.

A univariate frequency analysis is useful when one random variable is significant in the design process. The bivariate analysis of the return periods of flood volume and flood peak may however provide more useful information for design criteria than a univariate analysis [4] if more than one variable is significant in the design process of a hydraulic structure [2]. The flood risk related to a specific event can be wrongly assessed if only the univariate return period of either the peak discharge or the flood volume is analyzed in a case where a bivariate analysis would be appropriate [6]. If two variables are significant in the design process, it is advisable to use the bivariate return period to determine the design variable quantiles. Depending on the problem at hand, one of the approaches to define a bivariate return period is chosen. The choice should be made with care and one should be aware that the approaches outlined in the previous sections provide different design variable quantiles.

The effect of the choice of one of the concepts introduced above on the design variable quantile is illustrated in the following paragraphs using a concrete example. The example shall raise the awareness of the importance of a good problem definition. As stressed by Serinaldi [2], the choice between the possible definitions depends on how the system under consideration responds to a specific forcing. The failure mechanism of interest has a unique probabilistic description that results in a specific type of probability which in turn corresponds to a unique definition of the return period.

EFFECT OF RETURN PERIOD CHOICE ON DESIGN VARIABLE QUANTILES

Based on the above, we can look at the effect of the choice of the return period definition, which depends on the problem at hand, on the design variable quantiles of flood peak and volume. We use flood events in a study catchment in Switzerland, the Birse at Moutier-la-Charrue, to illustrate the design variable quantiles resulting from different return period definitions. The Birse catchment lies in the Swiss Jura, has an area of 183 km^2, a mean elevation of 930 m.a.s.l., and no glaciers. The measurement station is situated in Moutier-la-Charrue at 519 m.a.s.l. The measurements started in 1974 and there is a runoff time series of more than 40 years available for the analysis.

Before calculating the bivariate return period, some preparation is necessary:

1. *Sampling of flood events*: We worked with the POT approach to select four flood events per year on average from the runoff time series. This has the advantage of having all important events in the data set, even those that are not annual maxima. However, we needed to be cautious that these events were independent from each other. This was ensured by prescribing a minimum time difference between two successive events.
2. *Baseflow separation*: It is important to distinguish between the slow and the fast runoff components to analyze the statistical properties of floods [31]. We applied a recursive digital filter to separate baseflow from quick flow. Recursive digital filters have been shown to be easily applicable to a wide variety of catchments and to provide reliable results [39].
3. *Determination of marginal distributions*: We need some information about the marginal distributions of flood peaks and volumes before we go into the bivariate analysis and consider their joint behavior. Therefore, we need to determine their marginal distributions. The events were chosen using a POT approach and therefore follow a GPD distribution according to extreme value theory [17]. Because the threshold was only applied to the peaks and not to the volumes, the volumes do not follow a GPD but a GEV distribution. The goodness-of-fit of the GPD to the peak discharges and the GEV to the flood volumes was assessed using two graphical goodness-of-fit tests, pp-plots and qq-plots, and the upper-tail Anderson Darling test proposed by Chernobai et al. [40] which showed good results. The parameters of the GEV and GPD distributions estimated for the Birse catchment are shown in Table 2.

Table 2: Parameters of the GEV and GPD distributions estimated for the Birse catchment at Moutier-la-Charrue.

Parameter/distribution	**GEV**	**GPD**

Location $(\boldsymbol{\mu_l})$	294.1	12.9
Scale $(\boldsymbol{\sigma})$	93.1	11.6
Shape $(\boldsymbol{\xi})$	0.21	-0.15

4. *Fitting of a copula model*: The dependence between peak discharges and volumes was assessed by an exploratory data analysis using K-plots and Chi-plots [23]. A copula can be used to model the dependence between the two variables, peak discharge (Q) and flood volume (V). The dependence between the two variables Q and V was tested graphically by plotting all pairs of Q and V and numerically by computing two measures of dependence, Kendall's tau and Spearman's rho. Six copula models of the Archimedean copula family as well as two copulas of the meta-elliptical family were fitted using a pseudo-likelihood estimation method and tested using both graphical approaches and a goodness-of-fit test based on the Cramér-von Mises statistic [23, 25]. A p-value for the Cramér-von Mises statistic of each copula was estimated using a bootstrap procedure [25]. The copulas which were not rejected at a level of significance of 0.05 in most catchments were found to be the Joe and the Gumbel copula. We decided to work with the Joe copula because it was rejected in fewer catchments than the Gumbel copula. The Joe copula is described by

$$C(u,v) = 1 - \left[(1-u)^{\theta} + (1-v)^{\theta} - (1-u)^{\theta}(1-v)^{\theta}\right]^{\frac{1}{\theta}}. \tag{30}$$

It takes θ parameters in $[1,\infty)$ [24] and is able to model the dependence in the data. The parameter θ for the Birse catchment at Moutier-la-Charrue was estimated to be 1.92.

Knowing the copula found to model the dependence between peak discharges and flood volumes well, we can calculate the bivariate return period chosen to be suitable for the analysis. Table 3 shows the design quantiles for a return period of 100 years obtained for the Birse catchment by applying the different approaches outlined above. The results of all approaches are visualized in Figure 3.

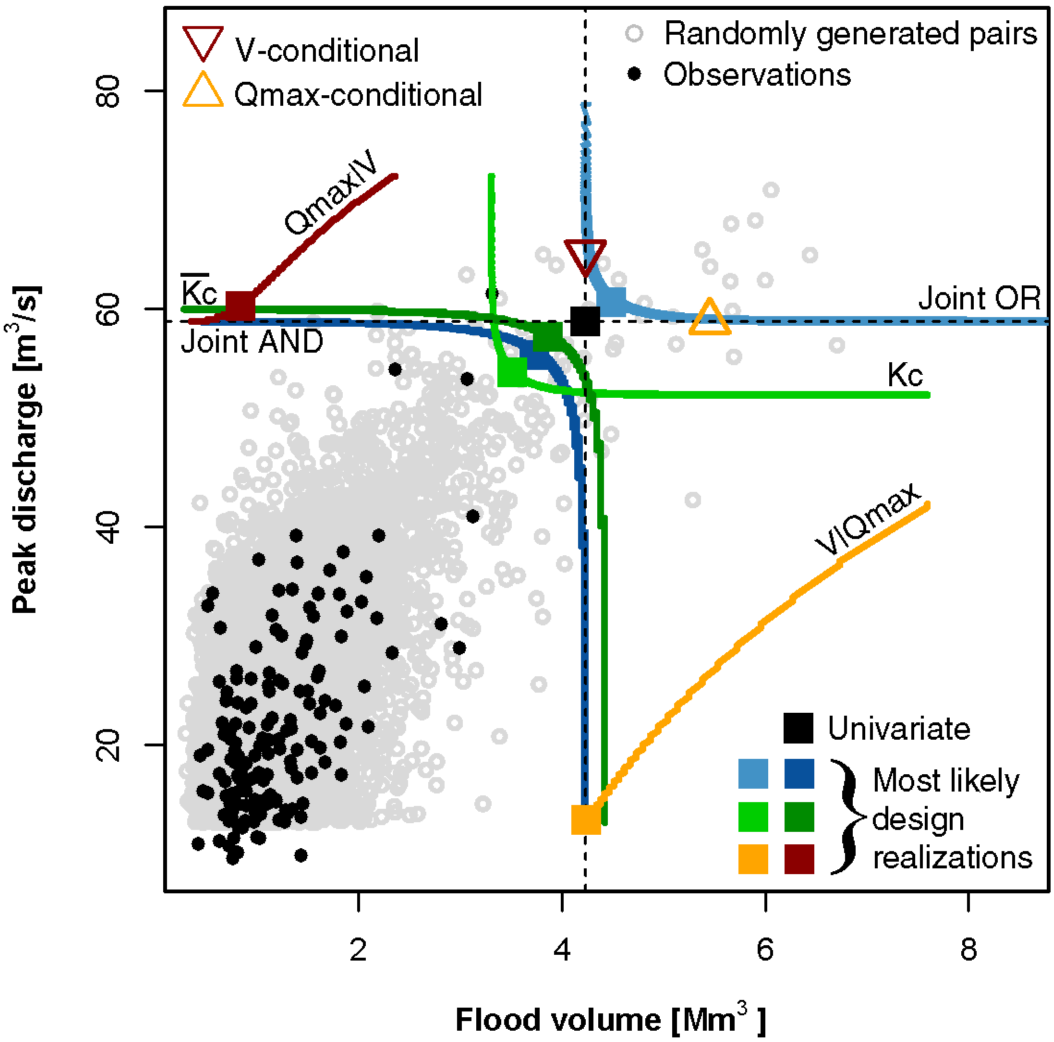


Figure 3: Design variable quantiles for different return period definitions. The black dots stand for the observed flood events for the Birse at Moutier-la-Charrue and the grey dots are 10 000 randomly generated pairs using the bivariate distribution of the peak discharges and flood volumes. The black square stands for the univariate quantile. The triangles represent the design variable pairs resulting from the Qmax-conditional and V-conditional approaches applied to the joint OR isoline. The isolines represent the return level curves for the two joint approaches AND and OR, and the approaches using the Kendall's and survival Kendall's distribution function. The squares on the isolines stand for the most-likely design realizations on these isolines.

Table 3: Design variable quantiles for the peak discharge and flood volume in the Birse catchment at Moutier-la-Charrue for the following approaches for a return period of 100 years: Univariate, Qmax|V, V|Qmax, Qmax-conditional, V-conditional, joint AND, joint OR, Kendall's, and survival Kendall's. If the approach provides us with an isoline, the most probable event on that isoline was chosen.

Approach	**Uni-variate**	**Qmax|V**	**V|Qmax**	**Qmax-condi-tional**	**V-condi-tional**	**Joint AND**	**Joint OR**	**Kendall's**	**Survival Kendall's**

Quantiles Q[m³/s]	58.8	60.2	13.1	58.9	65.1	55.8	60.6	54.2	57.4
Quantiles V[Mm³]	4.2	0.8	4.2	5.5	4.2	3.7	4.5	3.5	3.9

Conditional approach

We chose two different types of conditional events to illustrate the conditional approach (Equations 9 and 10). However, if desired, one could work with different types of conditional events such as $\{X > x | Y < y\}$ or $\{Y > y \mid X < x\}$. If the flood volume is considered to be the most significant variable for the design process, we work with the event given in Equation 9 and call the approach Qmax|V. On the contrary, if the peak discharge is considered to be most important, we work with the event given in Equation 10 and call the approach V|Qmax.

The design quantiles using these conditional approaches were calculated using Equations 15 and 16. We retained the pairs (u, v) that were located along the probability level t corresponding to the given return period T such that $1 - t = \frac{1-u-v+C(u,v)}{1-v}$ when looking at the first event and $1 - t = \frac{1-u-v+C(u,v)}{1-u}$ when looking at the second event.

All the pairs of probabilities (u, v) that are at the same probability level t are eligible because they correspond to the return period T. The design variable pairs were then calculated according to their marginal distributions F_Q and F_V

$$Q_T = F_Q^{-1}(u) \text{ and} \tag{31}$$

$$V_T = F_V^{-1}(v). \tag{32}$$

Therefore, in contrast to the univariate case, there is no unique solution of the design variables associated with the return period T. Instead, all the possible solutions are located along the return period level, which is a curve on a bi-dimensional graph with Q and V as coordinates.

Figure 3 shows the conditional return period levels for the two conditional approaches discussed above (Qmax|V and V|Qmax). If desired, one design variable pair on the isoline can be selected *e.g.* by choosing the most probable design realization (see Table 3 and squares on isoline in Figure 3).

Joint approaches

If the problem at hand requires a joint analysis of peak discharges and volumes, a joint approach is appropriate. The joint approach does not provide a single design quantile pair for a given return period, but a set of pairs, all having the same return period. As mentioned in the theoretical part, two possible approaches to compute a joint return period are the AND and the OR approach.

The design quantiles using the OR approach were calculated using Equation 21. Equation 22 was used in the AND approach. We retained the pairs (u, v) that were located along the probability level t

corresponding to the given return period T such that $1 - t = 1 - C(u, v)$ in the OR approach, and $1 - t = 1 - u - v + C(u, v)$ in the AND approach.

All the pairs of probabilities (u, v) that are at the same probability level t are eligible because they correspond to the return period T. The design variable pairs were then calculated according to their marginal distributions F_Q and F_V using Equations 31 and 32.

Therefore, in contrast to the univariate case, there is no unique solution of the design variables associated with the return period T. Instead, all the possible solutions are located along the return period level, which is a curve on a bi-dimensional graph with Q and V as coordinates.

Figure 3 shows the joint return period levels for the AND and OR approaches. While the joint return period level using the AND approach is concave, the joint return period level using the OR approach is convex. Generally, the AND approach provides lower design variable quantiles than the OR approach for a given return period. If desired, one design variable pair on the isoline can be selected *e.g.* by choosing the most-likely design realization (see Table 3 and squares on isoline in Figure 3) or by applying the H-conditional approach (here, the Qmax-conditional and V-conditional approaches) proposed by Salvadori et al. [12].

Kendall's approach

The design quantiles using the Kendall's approach were calculated using Equation 24. The Kendall's quantile q_t for the probability level t could then be computed as

$$q_t = {K_c}^{-1}(t), \tag{33}$$

where ${K_c}^{-1}$ is the inverse of the Kendall's distribution K_c. We estimated q_t using a bootstrap technique [5]. We retained the pairs (u, v) that are located along the critical probability level q_t. All the pairs of probabilities (u, v) that are at the same probability level q_t are eligible because they correspond to the return period T. The design variable pairs were then calculated according to their marginal distributions F_Q and F_V using Equations 31 and 32.

The isoline corresponding to the critical events according to the Kendall's return period is displayed in Figure 3. The event with the highest likelihood on this isoline is also indicated and given in **Error! Reference source not found.**.

Similary to the Kendall's distribution function, the survival Kendall's distribution function can be used to derive a survival Kendall's quantile instead of the Kendall's quantile

$$q_t = \overline{K_c}^{-1}(t), \tag{34}$$

where $\overline{K_c}^{-1}$ is the inverse of the Kendall's survival function [12, 34]. The isoline corresponding to the critical events according to the survival Kendall's return period is also displayed in **Error! Reference source not found.**3. The event with the highest likelihood on this isoline is indicated with a square and given in **Error! Reference source not found.**.

The results presented above for different approaches to compute design variable quantiles demonstrate that the choice of the approach has a significant influence on the outcome of the design variable quantiles and that it is therefore essential to well define the problem at hand to make a suitable choice of a return period definition. Compared to the univariate quantile, the choice of the joint OR approach resulted in higher design variable quantiles. In contrast, the choice of the conditional approaches, the joint AND approach, the Kendall's approach and the survival Kendall's approach resulted in lower design variable quantiles than in the univariate case. Serinaldi [2] emphasized that this choice is not arbitrary and depends on the problem at hand. If not only the problem at hand but also the interaction of the design variables X and Y with the structure under consideration is well defined, the structure-based return period introduced by Volpi and Fiori [35] can be applied to derive the design variable quantile

$$z_T = {F_Z}^{-1}\left(1 - \frac{\mu}{T}\right), \tag{35}$$

where ${F_Z}^{-1}$ is the inverse of the distribution function of the design parameter Z. In practice, the structure function $g(X, Y)$ relating the hydrological variables peak discharge and volume to the design parameter Z might be quite complex. For a specific example of a structure function, we refer to Volpi and Fiori [35] and to Salvadori et al. [36].

Choice of a point on the isoline

It was mentioned earlier on that there are several possibilities to choose one design realization on the return level curve or isoline. These possibilities are illustrated in Figure 4. The isoline can be divided into a central and a naïve part. The determination of the component-wise excess realization or the most-likely design realization is a way of choosing just one realization on the critical level. If one would not like to go that far, a possibility is to work with an ensemble sampled according to the probability distribution.

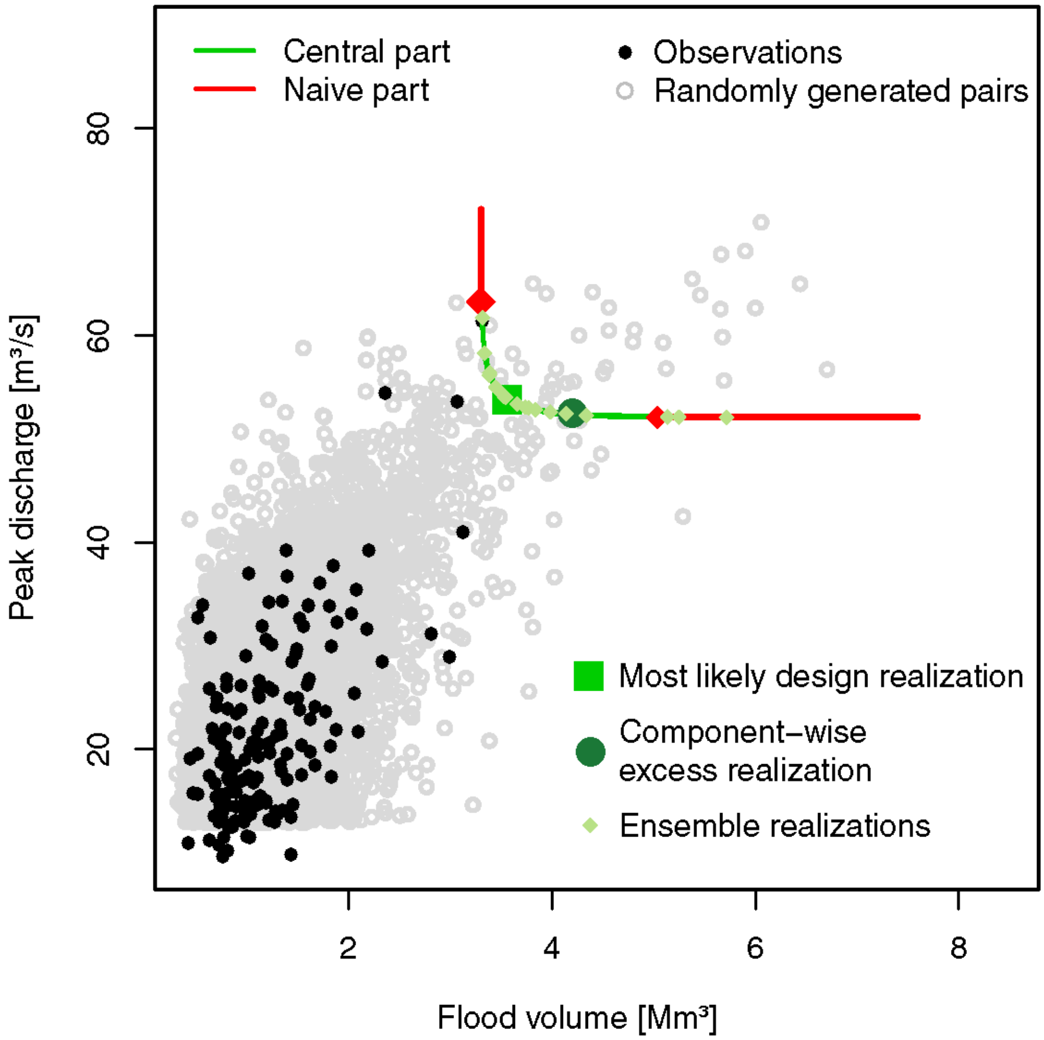


Figure 4: Kendall's critical level divided into a central (green) and two naïve parts (red). The black dots stand for the observed flood events for the Birse at Moutier-la-Charrue and the grey dots are 10 000 randomly generated pairs using the bivariate distribution of the peak discharges and flood volumes. The two possibilities of choosing one design realization are displayed. Namely, these are the most-likely design realization and the component-wise excess realization. It is also shown how a subset of realizations can be chosen with the ensemble approach.

Uncertainty of design variable quantiles

Independent of the return period definition used to estimate the design variable quantiles, the design variable quantiles have to be complemented with information about their uncertainty [41]. Estimated design variable quantiles are uncertain because they are made for events whose frequency goes beyond the range that is supported by the length of the flood records [42]. In a bivariate framework, the uncertainty related to the limited sample size and the uncertainty of the marginal distributions combine with the uncertainty of the dependence structure between the two variables [18]. The scientific community agrees that the uncertainty stemming from flood frequency analysis should be properly acknowledged not only in a univariate but also in a multivariate setting [12, 18, 36, 43, 44]. Serinaldi [43] recommended communicating the results of hydrological frequency analysis by complementing accurate point estimates with realistic confidence intervals. The uncertainty of extreme quantiles can be assessed using bootstrapping methods not only in univariate but also in multivariate frequency analysis [43]. Serinaldi [43] and Dung [44] proposed non-parametric and parametric bootstrap algorithms to construct confidence intervals for design variable quantiles. Practical procedures for assessing the uncertainty of multivariate design occurrences via bootstrap approaches have recently been outlined in Salvadori et al. [28].

Serinaldi [41] distinguished between three different types of uncertainty in a statistical analysis: natural uncertainty which represents the randomness and complexity of the natural process; statistical uncertainty which is related to the estimation of the parameters; and model uncertainty which depends on the choice of the statistical model. While natural uncertainty can not be reduced, statistical uncertainty can be reduced by increasing the sample size and model uncertainty by increasing the knowledge of the process under study. Serinaldi [43] and Dung [44] stated that the major source of uncertainty in the estimation of design variable quantiles is the limited sample size while parameter estimation methods and model selection are of only minor importance. When the sample is small, many joint distributions and copulas can fit the data adequately and goodness-of-fit tests cannot discriminate between alternative models because of the lack of power. Very large samples are needed to reliably estimate the marginal and joint extreme quantiles [43]. The uncertainty of design variable quantile estimates can be reduced by information expansion such as the inclusion of documentary records of historical floods or data pooling from similar catchments [42, 44]. Uncertainty can also be reduced using Bayesian techniques allowing for the incorporation of different sources of information [45].

DISCUSSION & CONCLUSIONS

The results presented above for different approaches to compute design variable quantiles demonstrated that the choice of the approach has a significant influence on the outcome of the design variable quantiles. The case study for a catchment in Switzerland showed that a univariate analysis can not provide a complete assessment of the probability of occurrence of a flood event if two or more dependent variables are significant in the design process. This confirms earlier results [6, 8], that univariate approaches might lead to an inadequate estimation of the risk associated with a given event. Given a specific problem, a solution to the problem of how to define a multivariate return period can be found [2]. The approaches of defining a return period discussed in this review, resulted in different design event estimates. This implies that addressing the question of how to specify the engineering problem and therewith to define a bivariate return period is important. It is impossible to provide the reader with a general suggestion for an approach to estimate multivariate design events since that depends on the problem he or she is facing. Still, this paper should give him or her an overview on the methods involved in defining a return period once he or she has outlined the problem. This paper provides a basis for the practitioner or engineer to decide which of the strategies to define a return period is most suitable in his or her case. In particular, the theoretical background of five different approaches to compute design variable quantiles using conditional and joint probabilities is described, and the challenge of defining a return period was discussed with respect to flood events looking at the two variables peak discharge and flood volume. However, the analysis is neither restricted to floods nor to two variables. The concepts discussed above are also applicable in a context where more than two dependent variables are important and in other areas of application. Though, recently, it has even been questioned whether the return period and the corresponding design quantiles do actually matter in system design and planning. Serinaldi [2] strongly recommended assessing the risk of failure instead, which is the probability to observe a critical event at least once in M years of the design life of a structure. The risk of failure has a unique definition independent of the nature of data and allows the consideration of both independent and dependent variables in stationary but also non-stationary settings. A multivariate failure approach to assess hydrological risk in a general and consistent mathematical way seems valuable and has recently been outlined by Salvadori et al. [28].

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REFERENCES

1. Gräler B, van den Berg MJ, Vandenberghe S, Petroselli A, Grimaldi S, Baets BD, Verhoest NEC. Multivariate return periods in hydrology: a critical and practical review focusing on synthetic design hydrograph estimation. *Hydrological Earth System Sciences* 2013, 17:1281-1296.
2. Serinaldi F. Dismissing return periods! *Stochastic Environmental Research and Risk Assessment* 2015, 29:1179-1189. doi:10.1007/s00477-014-0916-1.
3. Volpi E, Fiori A, Grimaldi S, Lombardo F, Koutsoyiannis D. One hundred years of return period: Strengths and limitations. *Water Resources Research* 2015, 51:8570-8585. doi:10.1002/2015WR017820.
4. Salvadori G, De Michele C. Frequency analysis via copulas: Theoretical aspects and applications to hydrological events. *Water Resources Research* 2004, 40. doi:10.1029/2004WR003133.
5. Salvadori G, DeMichele C, Durante F. On the return period and design in a multivariate framework. *Hydrological Earth System Sciences* 2011, 15:3293-3305.
6. Requena AI, Mediero L, Garrote L. A bivariate return period based on copulas for hydrologic dam design: accounting for reservoir routing in risk estimation. *Hydrological Earth System Sciences* 2013, 17:3023-3038.
7. Robson A, Reed D. *Statistical procedures for flood frequency estimation. Volume 3 of the Flood Estimation Handbook*. Wallingford UK: Centre for Ecology & Hydrology; 2008, 338.
8. Yue S, Rasmussen P. Bivariate frequency analysis: discussion of some useful concepts in hydrological application. *Hydrological Processes* 2002, 16:2881-2898.
9. Salvadori G. Bivariate return periods via 2-copulas. *Statistical Methodology* 2004, 1:129-144.
10. Salvadori G, De Michele C. Multivariate multiparameter extreme value models and return periods: A copula approach. *Water Resources Research* 2010, 46. doi:10.1029/2009WR009040.
11. Vandenberghe S, Verhoest NEC, Onof C, De Baets B. A comparative copula-based bivariate frequency analysis of observed and simulated storm events: A case study on bartlett-Lewis modeled rainfall. *Water Resources Research* 2011, 47. doi:10.1029/2009WR008388.
12. Salvadori G, Tomasicchio GR, D'Alessandro F. Practical guidelines for multivariate analysis and design in coastal and off-shore engineering. *Coastal Engineering* 2014, 88:1-14.
13. Chebana F, Ouarda TBMJ. Multivariate quantiles in hydrological frequency analysis. *Environmetrics* 2009, 22:63-78.
14. Hingray B, Picouet C, Musy C. *Hydrology. A science for engineers*. Lausanne: CRC Press; 2014, 592.

15. Weingartner R. Regionalhydrologische Analysen - Grundlagen und Anwendungen. *Beiträge zur Hydrologie der Schweiz*. Vol. 37. Bern: Schweizerische Gesellschaft für Hydrologie und Limnologie (SGHL); 1999, 178.
16. Lang M, Ouarda TBMJ, Bobée B. Towards operational guidelines for over-threshold modeling. *Journal of Hydrology* 1999, 225:103-117.
17. Coles S. *An introduction to statistical modeling of extreme values*. London: Springer; 2001, 208.
18. Serinaldi F. Can we tell more than we can know? The limits of bivariate drought analyses in the United States. *Stochastic Environmental Research and Risk Assessment* 2015. doi:10.1007/s00477-015-1124-3.
19. Kotz S, Balakrishnan N, Johnson NL. *Continuous Multivariate Distributions, Volume 1, Models and Applications, 2nd Edition*; 2000, 752.
20. Salvadori G, C. De Michele, Kottegoda NT, Rosso R. *Extremes in nature. An approach using copulas*. Vol. 56. Dordrecht: Springer; 2007, 292.
21. International Commission on Statistical Hydrology. Copula Function. Available at: http://www.stahy.org. (Accessed 20.04.2016)
22. Sklar A. Fonctions de répartition à n dimensions et leurs marges. *Publications de l'Institut de Statistique de l'Université de Paris* 1959, 8:229-231.
23. Genest C, Favre A-C. Everything you always wanted to know about copula modeling but were afraid to ask. *Journal of Hydrologic Engineering* 2007, 12:347-367.
24. Nelsen RB. *An introduction to copulas*. Second edition ed. New York: Springer Science & Business Media; 2005, 269.
25. Genest C, Rémillard B, Beaudoin D. Goodness-of-fit tests for copulas: A review and a power study. *Insurance: Mathematics and Economics* 2009, 44:199-213.
26. Meylan P, Favre A-C, Musy A. *Predictive hydrology. A frequency analysis approach*: Science Publishers. St. Helier, Jersey, British Channel Islands; 2012, 212.
27. Joe H. *Dependence modeling with copulas.* Vancouver: Chapman & Hall/CRC; 2014, 459.
28. Salvadori G, Durante F, Michele CD, Bernardi M, Petrella L. A multivariate copula-based framework for dealing with hazard scenarios and failure probabilities. *Water Resources Research* 2016, 52:3701-3721. doi:10.1002/2015WR017225.
29. Renard B, Lang M. Use of a Gaussian copula for multivariate extreme value analysis: Some case studies in hydrology. *Advances in Water Resources* 2007, 30:897-912.
30. Durante F, Salvadori G. On the construction of multivariate extreme value models via copulas. *Environmetrics* 2010, 21:143-161.
31. Yue S, Ouarda TBMJ, Bobée B, Legendre P, Bruneau P. Approach for describing statistical properties of flood hydrograph. *Journal of Hydrologic Engineering* 2002, 7:147-153.
32. Shiau JT. Return period of bivariate distributed extreme hydrological events. *Stochastic Environmental Research and Risk Assessment* 2003, 17:42-57.
33. Ghoudi K, Khoudraji A, Rivest EL-P. Propriétés statistiques des copules de valeurs extrêmes bidimensionnelles. *Canadian Journal of Statistics* 1998, 26:187-197. doi:10.2307/3315683.
34. Salvadori G, Durante F, Michele CD. Multivariate return period calculation via survival functions. *Water Resources Research* 2013, 49:2308-2311. doi:10.1002/wrcr.20204, 2013.
35. Volpi E, Fiori A. Hydraulic structures subject to bivariate hydrological loads: Return period, design, and risk assessment. *Water Resources Research* 2014, 50:885-897. doi:10.1002/2013WR014214.
36. Salvadori G, Durante F, Tomasicchio GR, D'Alessandro F. Practical guidelines for the multivariate assessment of the structural risk in coastal and off-shore engineering. *Coastal Engineering* 2015, 95:77-83.

37. Salvadori G, Michele CD. Chapter 5. Multivariate extreme value methods. In: AghaKouchak A, Easterling D, Hsu K, Schubert S, Sorooshian S, eds. *Extremes in a changing climate. Detection, analysis and uncertainty*. Dordrecht: Springer Science+Business Media; 2013, 115-162.
38. Volpi E, Fiori A. Design event selection in bivariate hydrological frequency analysis. *Hydrological Sciences Journal* 2012, 57:1506-1515. doi:10.1080/02626667.2010.726357.
39. Eckhardt K. How to construct recursive digital filters for baseflow separation. *Hydrological Processes* 2005, 19:507-515.
40. Chernobai A, Rachev ST, Fabozzi FJ. Composite goodness-of-fit tests for left-truncated loss samples. In: Lee C-F, Lee J, eds. *Handbook of financial econometrics and statistics*. New York: Springer Science+Business Media; 2015, 575-596.
41. Serinaldi F. Assessing the applicability of fractional order statistics for computing confidence intervals for extreme quantiles. *Journal of Hydrology* 2009, 376:528-541.
42. Reed DW. Reinforcing flood-risk estimation. *Philosophical Transactions of the Royal Society of London* 2002, 360:1373-1387.
43. Serinaldi F. An uncertain journey around the tails of multivariate hydrological distributions. *Water Resources Research* 2013, 49:6527-6547. doi:10.1002/wrcr.20531, 2013.
44. Dung NV. Handling uncertainty in bivariate quantile estimation - An application to flood hazard analysis in the Mekong Delta. *Journal of Hydrology* 2015, 527:704-717. doi:10.1016/j.jhydrol.2015.05.033.
45. Zhang Q, Xiao M, Singh VP. Uncertainty evaluation of copula analysis of hydrological droughts in the East River basin, China. *Global and Planetary Change* 2015, 129:1-9. doi:10.1016/j.gloplacha.2015.03.001.